Int J Theor Phys (2009) 48: 815–824
DOI 10.1007/s10773-008-9855-2

Casimir Effect for Concentric Spheres in de Sitter Spacetime with Signature Change

F. Darabi

Received: 18 July 2008 / Accepted: 15 September 2008 / Published online: 1 October 2008

Abstract We revisit the Casimir effect for two concentric spherical shells in de Sitter background with a new geometric configuration, namely Euclidean signature between and Lorentzian signature outside the spheres with different cosmological constants, for a massless scalar field satisfying Dirichlet boundary conditions on the spheres. It is shown that an extra constant pressure emerges due to this signature changing configuration. Some interesting aspects of this extra term are then discussed.

Keywords Casimir effect · Signature change · de Sitter spacetime

1 Introduction

The Casimir effect is regarded as one of the most striking manifestation of vacuum fluctuations in quantum field theory. The presence of reflecting boundaries alters the zero-point modes of a quantized field, and results in the shifts in the vacuum expectation values of quantities quadratic in the field, such as the energy density and stresses. Therefore, the Casimir effect can be viewed as the polarization of the vacuum by boundary conditions or geometry. In particular, vacuum forces arise acting on the constraining boundaries. The particular features of these forces depend on the nature of the quantum field, the type of spacetime manifold and its dimensionality, the boundary geometries and the specific boundary conditions imposed on the field. Since the original work by Casimir in 1948 [1] many theoretical and experimental works have been done on this problem [2–15].

The time dependence of boundary conditions or geometries, the so-called dynamical Casimir effect, is also a new element which has to be taken into account. In particular, in [16] the Casimir effect has been calculated for a massless scalar field satisfying Dirichlet

F. Darabi (✉)
Department of Physics, Azarbaijan University of Tarbiat Moallem, Tabriz, 53714-161, Iran
e-mail: f.darabi@azaruniv.edu

F. Darabi
Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, 55134-441, Iran

boundary conditions on the spherical shell in de Sitter space. The Casimir stress is calculated for inside and outside of the shell with different backgrounds corresponding to different cosmological constants.

On the other hand, signature changing spacetimes have recently been of particular importance as specific geometries with interesting physical effects. The initial idea of signature change is due to Hartle, Hawking and Sakharov [17, 18] which makes it possible to have a spacetime with Euclidean and Lorentzian regions in quantum gravity. It has been shown that the signature change may happen even in classical general relativity [19–29]. The issue of propagation of quantum fields on signature-changing spacetimes has also been of some interest [30–35]. For example, Dray et al. have shown that the phenomenon of particle production can happen for propagation of scalar particles in spacetime with heterotic signature. They have also obtained a rule for propagation of massless scalar fields on a two dimensional spacetime with signature change.

The Casimir effect has also recently been studied in a signature changing two dimensional spacetime, having a cylindrical topology with a narrow Euclidean region encircled by outside Lorentzian signature, and shown that there is a non-vanishing pressure on the hypersurface of signature change [36]. The Casimir stress on a spherical shell in de Sitter signature changing background for massless scalar field satisfying Dirichlet boundary conditions on the shell is also calculated [37]. Motivated by this new element in studying the Casimir effect we have paid attention to study such a non-trivial effect in a model of two cocentric spherical shells in de Sitter space with different cosmological constants and metric signatures, namely Euclidean signature between and Lorentzian one outside the spheres. The Casimir stress on two cocentric spherical shells in de Sitter spacetime with fixed Lorentzian background has already been obtained in [38]. In this paper, we will generalize this result to a signature changing background.

2 Casimir Effect for Concentric Spheres in Flat Space-Time

In this section we shall use the results of [39]. Consider two concentric spherical shells with zero thickness and radii a and b, $a < b$. Consider now the Casimir force due to fluctuation of a free massless scalar field satisfying Dirichlet boundary conditions on the spherical shells in Lorentzian space-time. The vacuum force per unit area of the inner sphere is given by

$$F_a(a,b) = F(a) - P_a(a,b,a), \tag{1}$$

where $F(a)$ is the force per unit area of a single sphere with radius a, and $P_a(a,b,a)$ is due to the existence of the second sphere (interaction force). Similarly, the vacuum force acting per unit area of outer sphere is

$$F_b(a,b) = F(b) + P_b(a,b,b). \tag{2}$$

Therefore, the vacuum force per unit area of a single sphere is the sum of Casimir forces F_{in} and F_{out} for inside and outside of the shell.

$$F(a) = F_{in}(a) + F_{out}(a), \qquad F(b) = F_{in}(b) + F_{out}(b). \tag{3}$$

Casimir forces inside and outside of the shell are divergent individually [40], but when we add interior and exterior forces to each other, divergent parts cancel each other out.

Interaction forces $P_a(a,b,a)$ and $P_b(a,b,b)$ are finite and for Dirichlet boundary condition, are given by

$$P_a(a,b,a) = \frac{-1}{8\pi^2 a^3}\sum_{l=0}^{\infty}(2l+1)\int_0^{\infty} dz \frac{K_\nu^{(b)}(bz)/K_\nu^{(a)}(az)}{K_\nu^{(a)}(az)I_\nu^{(b)}(bz) - K_\nu^{(b)}(bz)I_\nu^{(a)}(az)}, \quad (4)$$

$$P_b(a,b,b) = \frac{-1}{8\pi^2 b^3}\sum_{l=0}^{\infty}(2l+1)\int_0^{\infty} dz \frac{I_\nu^{(a)}(az)/I_\nu^{(b)}(bz)}{K_\nu^{(a)}(az)I_\nu^{(b)}(bz) - K_\nu^{(b)}(bz)I_\nu^{(a)}(az)}, \quad (5)$$

where I_ν and K_ν are modified Bessel function.

3 Casimir Effect for a Sphere in de Sitter Space

We will consider a conformally coupled massless scalar field ϕ satisfying the following Dirichlet boundary condition on a spherical shell in de Sitter space

$$\phi(x=a) = 0, \quad (6)$$

where a is the radius of spherical shell. The corresponding field equation has the form

$$(\nabla_\mu \nabla^\mu + \xi R)\phi = 0, \quad \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu), \quad (7)$$

where ∇_μ is the covariant derivative operator, and R is the Ricci scalar for de Sitter space. In the conformally coupled case, the corresponding stress-energy tensor is defined as [2]

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - 1/2)g_{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi + \frac{1}{12}g_{\mu\nu}\phi\nabla_\lambda\nabla^\lambda\phi, \quad (8)$$

where $\xi = \frac{1}{6}$. To make maximum use of the flat-space calculations we use de Sitter metric in conformally flat form

$$ds^2 = \Omega^2(\eta)\left[d\eta^2 - \sum_{i=1}^{3}(dx^i)^2\right], \quad (9)$$

where $\Omega(\eta) = \frac{\alpha}{\eta}$ and η is the conformal time [2]

$$-\infty < \eta < 0, \quad (10)$$

which covers half of de Sitter space conformal to a portion of Minkowski space. Notice that we assume the same coordinate system and the same range for the values of coordinates in two conformally related metrics. In particular, if we take from the one side a spherical shell in the Minkowski spacetime with the time variable $-\infty < \eta < \infty$, in the corresponding problem in de Sitter spacetime the range for η should be the same. However, since the metric (9) is symmetric under $\eta \to -\eta$ and we are interested in a cosmology with forward in time evolution, we then consider the range $0 < \eta < \infty$ for this metric to cover the other half.

The quantization of a scalar field on background of the metric (9) is straightforward. Let $\{\phi_k(x), \phi_k^\star(x)\}$ be a complete set of orthonormalized positive and negative frequency solutions to the field equation (7), obeying boundary condition (6). By expanding the field operator over these eigenfunctions, using the standard commutation rules and the definition of the vacuum state for the vacuum expectation values of the energy-momentum tensor one obtains [2]

$$\langle 0|T_\mu^\nu|0\rangle = \sum_k T_\mu^\nu\{\phi_k, \phi_k^*\}, \tag{11}$$

where $|0\rangle$ is the amplitude for the corresponding vacuum state, and the bilinear form $T_\mu^\nu\{\phi, \phi^*\}$ on the right is determined by the classical energy-momentum tensor (8). In the problem under consideration we have a conformally trivial situation, namely a conformally invariant field on background of the conformally flat spacetime. Instead of evaluating (11) directly on background of the curved metric, the vacuum expectation values can be obtained from the corresponding flat spacetime results for a scalar field ϕ by using the conformal properties of the problem under consideration. Under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ the ϕ field will be changed by the rule

$$\tilde{\phi}(x) = \Omega^{-1}\phi(x). \tag{12}$$

Given two conformally related metrics $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, the corresponding energy-momentum tensor for conformally coupled situations are related as [2]

$$\langle 0|T_\mu^\nu[\tilde{g}_{kl}]|0\rangle|_{ren} = (g/\tilde{g})^{1/2}\langle 0|T_\mu^\nu[g_{kl}]|0\rangle|_{ren} - \frac{1}{2880\pi^2}\left[\frac{1}{6}{}^{(1)}\tilde{H}_\mu^\nu - {}^{(3)}\tilde{H}_\mu^\nu\right], \tag{13}$$

where g and $\tilde{g}$ are the determinants of the corresponding metrics. Now $\langle 0|T_\mu^\nu[g_{kl}]|0\rangle|_{ren}$, is the regularized energy-momentum tensor for a conformally coupled scalar field in the case of a spherical shell in flat spacetime. The second term in (13) is the vacuum polarization due to the gravitational field, without any boundary condition. The functions ${}^{(1,3)}\tilde{H}_\mu^\nu$ are some combinations of curvature tensor components [2]. The radial Casimir force per unit area $\frac{F}{A}$ on the sphere, called Casimir stress, is obtained from the radial-radial component of the vacuum expectation value of the stress-energy tensor:

$$\frac{F}{A} = \langle 0|T_{(in)}{}_r^r - T_{(out)}{}_r^r|0\rangle|_{x=a}. \tag{14}$$

4 Casimir Effect for Concentric Spheres in de Sitter Space with Different Signatures

Consider the system of two concentric spherical shells with zero thickness and with radii a and b, $(a < b)$ in de Sitter space. We assume different vacua in between and outside the spheres, corresponding to different α_{bet} and α_{out}. We also assume Euclidean signature for the region between the spheres, and Lorentzian signature for the region outside the spheres. We will use the de sitter metric in conformally flat form (9). The relation between parameter α and cosmological constant Λ is given by

$$\alpha^2 = \frac{3}{\Lambda}. \tag{15}$$

The energy for each single sphere is then obtained as

$$\bar{E}(a) = \frac{\eta^2}{6a}\left[(c_1\Lambda_{out} + c_2\Lambda_{bet}) + \frac{c_1'}{\varepsilon}(\Lambda_{out} - \Lambda_{bet})\right], \tag{16}$$

$$\bar{E}(b) = \frac{\eta^2}{6b}\left[(c_1\Lambda_{bet} + c_2\Lambda_{out}) + \frac{c_1'}{\varepsilon}(\Lambda_{bet} - \Lambda_{out})\right], \tag{17}$$

for which the renormalization leads to the Casimir energies [38]

$$\bar{E}_{ren}(a) = \frac{\eta^2}{6a}(c_1\Lambda_{out} + c_2\Lambda_{bet}), \tag{18}$$

$$\bar{E}_{ren}(b) = \frac{\eta^2}{6b}(c_1\Lambda_{bet} + c_2\Lambda_{out}), \tag{19}$$

where $c_1 = 0.008873$, $c_2 = -0.003234$, $c_1' = 0.001010$. Now we use the following relation for the stress on a shell in the Lorentzian metric

$$\left(\frac{\bar{F}(a)}{A}\right)_L = \frac{-1}{4\pi a^2}\frac{\partial \bar{E}}{\partial a}. \tag{20}$$

In the signature changing case considered here we have $\alpha_{in} \equiv \alpha_{out}$ and $\alpha_{out} \equiv \alpha_{bet}$ for the sphere with radius a, and $\alpha_{in} \equiv \alpha_{bet}$ and $\alpha_{out} \equiv \alpha_{out}$ for the sphere with radius b. Correspondingly we have

$$\left(\frac{\bar{F}(a)}{A}\right)_{L-E} = [\langle 0|T_r^r|0\rangle|_{out}^L - \langle 0|T_r^r|0\rangle|_{bet}^E], \tag{21}$$

$$\left(\frac{\bar{F}(b)}{A}\right)_{E-L} = [\langle 0|T_r^r|0\rangle|_{bet}^E - \langle 0|T_r^r|0\rangle|_{out}^L]. \tag{22}$$

The scalar field $\phi(\vec{x}, \eta)$ in the Lorentzian de Sitter space satisfies

$$(\Box + \xi R)\phi(\vec{x}, \eta) = 0, \tag{23}$$

where $\Box$ is the Laplace-Beltrami operator for de Sitter metric, and ξ is the coupling constant. For conformally coupled field in four dimension $\xi = \frac{1}{6}$, and R, the Ricci scalar curvature, is given by

$$R = 12\alpha^{-2}. \tag{24}$$

Taking into account the separation of variables as

$$\phi_L(r, \theta, \eta) = A(r)B(\theta)T_L(\eta), \tag{25}$$

for the Lorentzian region with

$$T_L(\eta) = \frac{1}{\sqrt{2\omega(2\pi)^3}}\exp^{-i\omega\eta}, \tag{26}$$

the corresponding Euclidean η-dependent term takes on the form

$$T_E(\eta) = \frac{1}{\sqrt{2\omega(2\pi)^3}}\exp^{-\omega\eta}, \tag{27}$$

so that the scalar field be normalized in η. It is easily expected that the only difference between $(\frac{\bar{F}}{A})_{L-E}$, $(\frac{\bar{F}}{A})_{E-L}$ in one hand, and $(\frac{\bar{F}}{A})_L$ on the other hand, which was obtained in

[16], lies in the calculations for $T_L(\eta)$ and $T_E(\eta)$. Based on the following mode expansion

$$\phi=\sum_i (a_i^- \varphi_i^- + a_i^+ \varphi_i^+), \tag{28}$$

and normal ordering $\langle 0|a_i^- a_i^+|0\rangle = 1$, the detailed calculations, using (11), (26) and (27) in (21) and (22), lead to the stresses on each single shell due to the boundary conditions

$$\begin{aligned}\left(\frac{\bar F(a)}{A}\right)_{L-E} &= \frac{-1}{4\pi a^2}\frac{\partial \bar E(a)}{\partial a} = \frac{\eta^2}{24\pi a^4}(c_1\Lambda_{out}+c_2\Lambda_{bet})\\ &\quad\times\left(1+\frac{1}{64\pi^5\eta^2}(\Lambda_{out}^{-1}-\Lambda_{bet}^{-1})\zeta(2)\right),\end{aligned} \tag{29}$$

$$\begin{aligned}\left(\frac{\bar F(b)}{B}\right)_{E-L} &= \frac{-1}{4\pi b^2}\frac{\partial \bar E(b)}{\partial b} = \frac{\eta^2}{24\pi b^4}(c_1\Lambda_{bet}+c_2\Lambda_{out})\\ &\quad\times\left(1+\frac{1}{64\pi^5\eta^2}(\Lambda_{bet}^{-1}-\Lambda_{out}^{-1})\zeta(2)\right),\end{aligned} \tag{30}$$

where $\zeta(2)$ is the Zeta function and Abel-Plana summation formula has been used to regularize the infinite sum $\sum_i \omega_i$.

Under conformal transformation, interaction forces are given by

$$\bar P_a(a,b,a)=\frac{\eta^2\Lambda_{bet}}{3}P_a(a,b,a), \qquad \bar P_b(a,b,b)=\frac{\eta^2\Lambda_{bet}}{3}P_b(a,b,b). \tag{31}$$

Therefore the total stress on the spheres due to boundary conditions are obtained

$$\begin{aligned}\frac{\bar F_a(a,b)}{A} &= \frac{\bar F(a)}{A}-\bar P_a(a,b,a)\\ &= \frac{\eta^2}{24\pi a^4}(c_1\Lambda_{out}+c_2\Lambda_{bet})+\frac{1}{1536\pi^6 a^4}(\Lambda_{out}^{-1}-\Lambda_{bet}^{-1})(c_1\Lambda_{out}+c_2\Lambda_{bet})\zeta(2)\\ &\quad-\frac{\eta^2\Lambda_{bet}}{3}P_a(a,b,a),\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{\bar F_b(a,b)}{B} &= \frac{\bar F(b)}{B}+\bar P_b(a,b,b)\\ &= \frac{\eta^2}{24\pi b^4}(c_1\Lambda_{bet}+c_2\Lambda_{out})+\frac{1}{1536\pi^6 a^4}(\Lambda_{bet}^{-1}-\Lambda_{out}^{-1})(c_1\Lambda_{bet}+c_2\Lambda_{out})\zeta(2)\\ &\quad+\frac{\eta^2\Lambda_{bet}}{3}P_b(a,b,b).\end{aligned} \tag{33}$$

Now, we obtain the pure effect of vacuum polarization due to the gravitational field without any boundary conditions in Euclidean region with the following metric

$$ds^2=-\Omega^2(\eta)\left[d\eta^2+\sum_{i=1}^{3}(dx^i)^2\right]. \tag{34}$$

To this end, we calculate the renormalized stress tensor for the massless scalar field in de Sitter space with Euclidean signature. We use (13), then we obtain $\langle 0|T_\mu^\nu[g_{kl}]|0\rangle|_{ren} = 0$, and

$$^{(1)}H_\mu^\nu = 0,$$

$$^{(3)}H_\mu^\nu = \frac{3}{\alpha^4}\delta_\mu^\nu.$$

Therefore

$$\langle 0|T_\mu^\nu[\tilde{g}_{kl}]|0\rangle|_{ren} = \frac{1}{960\pi^2\alpha^4}\delta_\mu^\nu, \tag{35}$$

which is exactly the same result for the Lorentzian case [2]. The corresponding effective radial pressures for the Euclidean (between) and Lorentzian (outside) regions with α_{bet} and α_{out}, due to pure effect of gravitational vacuum polarization without any boundary condition, are given respectively by

$$P_{bet}^E = -\langle 0|T_r^r[\tilde{g}_{kl}]|0\rangle|_{ren} = -\frac{1}{960\pi^2\alpha_{bet}^4},$$

$$P_{out}^L = -\langle 0|T_r^r[\tilde{g}_{kl}]|0\rangle|_{ren} = -\frac{1}{960\pi^2\alpha_{out}^4}.$$

The corresponding gravitational pressure on a single spherical shell is then given by

$$P_g = P_{in} - P_{out} = -\frac{1}{960\pi^2}\left(\frac{1}{\alpha_{in}^4} - \frac{1}{\alpha_{out}^4}\right). \tag{36}$$

Therefore, gravitational pressures over each sphere, are given by

$$P_g(a) = \frac{-1}{8640\pi^2}(\Lambda_{out}^2 - \Lambda_{bet}^2), \qquad P_g(b) = \frac{-1}{8640\pi^2}(\Lambda_{bet}^2 - \Lambda_{out}^2). \tag{37}$$

The total stress on each spherical shell due to pure gravitational and boundary effects, is then obtained

$$\begin{aligned} P_{tot}(a) = {} & \frac{\eta^2}{24\pi a^4}(c_1\Lambda_{out} + c_2\Lambda_{bet}) + \frac{1}{1536\pi^6 a^4}(\Lambda_{out}^{-1} - \Lambda_{bet}^{-1})(c_1\Lambda_{out} + c_2\Lambda_{bet})\zeta(2) \\ & - \frac{1}{8640\pi^2}(\Lambda_{out}^2 - \Lambda_{bet}^2) - \frac{\eta^2\Lambda_{bet}}{3}P_a(a,b,a), \end{aligned} \tag{38}$$

$$\begin{aligned} P_{tot}(b) = {} & \frac{\eta^2}{24\pi b^4}(c_1\Lambda_{bet} + c_2\Lambda_{out}) + \frac{1}{1536\pi^6 b^4}(\Lambda_{bet}^{-1} - \Lambda_{out}^{-1})(c_1\Lambda_{bet} + c_2\Lambda_{out})\zeta(2) \\ & - \frac{1}{8640\pi^2}(\Lambda_{bet}^2 - \Lambda_{out}^2) + \frac{\eta^2\Lambda_{bet}}{3}P_b(a,b,b). \end{aligned} \tag{39}$$

5 Discussion

We may discuss on different possible cases for the total stresses. Let us first consider $P_{tot}(a)$ and assume $\Lambda_{bet} > \Lambda_{out}$ with

$$c_1\Lambda_{out} + c_2\Lambda_{bet} > 0, \tag{40}$$

which leads the three first terms in $P_{tot}(a)$ to be positive. Also, noting that $P_a(a,b,a) < 0$, the forth term in (38) is always positive, therefore the total pressure $P_{tot}(a)$ on the inner shell is repulsive.

For the case $\Lambda_{out} > \Lambda_{bet}$ the first and forth time dependent terms are positive while the second and third constant terms are negative. Therefore the total pressure $P_{tot}(a)$ may be either negative or positive. Given $P^{tot}(a) < 0$ initially ($\eta = 0$), the first and forth terms will dominate after some time and the pressure changes to be positive, namely a repulsive one. The situation for $P_{tot}(b)$ is as follows. Consider

$$c_1 \Lambda_{bet} + c_2 \Lambda_{out} > 0. \tag{41}$$

For $\Lambda_{bet} > \Lambda_{out}$, all terms but the first one are negative. At $\eta = 0$, the pressure $P_{tot}(b)$ is negative, but at sufficiently late times there is a competition between the first positive and the forth negative time dependent terms, according to which the outer sphere will experience a repulsive or an attracting force, respectively. For the case $\Lambda_{out} > \Lambda_{bet}$, all the terms but the forth one are positive. At $\eta = 0$, the pressure $P_{tot}(b)$ is positive, but at sufficiently late times there is again a competition between the first positive and the forth negative time dependent terms, according to which the outer sphere will experience a repulsive or an attracting force, respectively.

Now, we consider the case

$$c_1 \Lambda_{out} + c_2 \Lambda_{bet} < 0, \tag{42}$$

with $\Lambda_{bet} > \Lambda_{out}$. In this case, the first and second terms are negative whereas the third and forth terms in $P_{tot}(a)$ are positive. At $\eta = 0$, the pressure $P_{tot}(a)$ may be positive or negative and at late times there is a competition between the first negative and the forth positive time dependent terms, according to which the outer sphere will experience an attracting or a repulsive force. For $\Lambda_{out} > \Lambda_{bet}$, the first and third terms are negative and the second and forth terms are positive. At $\eta = 0$, the pressure $P_{tot}(a)$ may be negative or positive and at late times there is a competition between the first negative and the forth positive time dependent terms, according to which the outer sphere will experience an attracting or a repulsive force.

In this case, the pressure $P_{tot}(b)$ behaves as follows. Consider

$$c_1 \Lambda_{bet} + c_2 \Lambda_{out} < 0. \tag{43}$$

For $\Lambda_{bet} > \Lambda_{out}$, all the terms but the second one are negative. At $\eta = 0$, the pressure $P_{tot}(b)$ may be positive or negative according to the second and third terms. For late times, however, the pressure becomes negative leading to an attracting force. For $\Lambda_{bet} < \Lambda_{out}$, all the terms but the third one are negative. At $\eta = 0$, the pressure $P_{tot}(b)$ may be negative or positive according to the second and third terms. For late times, however, the pressure becomes negative leading to an attracting force.

6 Conclusion

We have revisited two concentric spherical shells in a signature changing de Sitter metric conformally coupled to a massless scalar field, satisfying Dirichlet boundary conditions, and different cosmological terms inside and outside the spheres. We have obtained time independent extra terms for the total pressures on each sphere. These terms vanish for the special case of identical cosmological constants, as it happens for the gravitational pressures.

For different cosmological constants, however, the introduction of an Euclidean signature between the spheres leads to an extra constant term on each sphere.

Given a false vacuum Λ_{bet} between and a true vacuum Λ_{out} outside the spheres, namely $\Lambda_{bet} > \Lambda_{out}$, the constant terms arising from signature change contribute positively (negatively) to the total pressure for the inner (outer) sphere, compared with the pure Lorentzian case. Therefore, in this case there is an extra attraction between the two spheres. This attraction leads the Euclidean region to become smaller and be replaced by Lorentzian region.

On the contrary, given a true vacuum Λ_{bet} between and a false vacuum Λ_{out} outside the spheres, namely $\Lambda_{out} > \Lambda_{bet}$, the constant terms arising from signature change contribute negatively (positively) to the total pressure for the inner (outer) sphere, compared with the pure Lorentzian case. Therefore, in this case there is an extra repulsion between the two spheres. This repulsion leads the Lorentzian regions to become smaller and be replaced by Euclidean region.

We then come to the conclusion that in both cases the region endowed by the true vacuum is capable of extending to those regions of false vacuum. This may be of particular importance in the study of inflationary models of the early universe. If we assume there were some regions of Euclidean signature within spherical layers (endowed by a false vacuum) inside the bubbles with Lorentzian signature (endowed by a true vacuum), then these regions tend to be removed and replaced by Lorentzian regions, due to the Casimir effect.

Acknowledgement This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM).

References

1. Casimir, H.B.G.: Proc. K. Ned. Akad. Wet. **51**, 793 (1948)
2. Birrel, N.D., Davies, P.C.W.: Quantum Fields in Curved Space. Cambridge University Press, Cambridge (1982)
3. Mostepanenko, V.M., Trunov, N.N.: The Casimir Effect and Its Applications. Clarendon, Oxford (1997)
4. Plunien, G., Muller, B., Greiner, W.: Phys. Rep. **134**, 87 (1986)
5. Lamoreaux, S.K.: Am. J. Phys. **67**, 850 (1999)
6. Bordag, M., Mohidden, U., Mostepanenko, V.M.: Phys. Rep. **353**, 1 (2001)
7. Kirsten, K.: Spectral Functions in Mathematics and Physics. CRC Press, Boca Raton (2001)
8. Milton, K.A., DeRaad, L.L., Schwinger, J.: Ann. Phys. (N.Y.) **115**, 338 (1978)
9. Setare, M.R., Saharian, A.A.: Int. J. Mod. Phys. A **16**, 1463 (2001)
10. Setare, M.R.: Class. Quantum Gravity **18**, 2097 (2001)
11. Setare, M.R., Mansouri, R.: Class. Quantum Gravity **18**, 2659 (2001)
12. Setare, M.R.: Class. Quantum Gravity **18**, 4823 (2001)
13. Saharian, A.A., Setare, M.R.: Phys. Lett. B **552**, 119 (2003)
14. Saharian, A.A., Setare, M.R.: Phys. Lett. B **584**, 306 (2004)
15. Saharian, A.A., Setare, M.R.: Nucl. Phys. B (2005), accepted for publication
16. Setare, M.R., Mansouri, R.: Class. Quantum Gravity **18**, 2331 (2001)
17. Hartle, J.B., Hawking, S.W.: Phys. Rev. D. **28**, 2960 (1983)
18. Sakharov, A.D.: Sov. Phys. JETP. **60**, 214 (1984)
19. Ellis, G.F.R., Sumruk, A., Coule, D., Hellaby, C.: Class. Quantum Gravity **9**, 1535 (1992)
20. Hayward, S.A.: Class. Quantum Gravity **9**, 1851 (1992)
21. Hayward, S.A.: Class. Quantum Gravity **10**, L7 (1993)
22. Hayward, S.A.: Phys. Rev. D. **52**, 7331 (1995)
23. Dereli, T., Tucker, R.W.: Class. Quantum Gravity **10**, 365 (1993)
24. Kossowski, M., Kriele, M.: Proc. R. Soc. Lond. A **446**, 115 (1995)
25. Kossowski, M., Kriele, M.: Class. Quantum Gravity **10**, 1157 (1993a)
26. Kossowski, M., Kriele, M.: Class. Quantum Gravity **10**, 2363 (1993b)
27. Hellaby, C., Dray, T.: Phys. Rev. D. **49**, 5096 (1994)
28. Hellaby, C., Dray, T.: J. Math. Phys. **35**, 5922 (1994)

29. Hellaby, C., Dray, T.: Phys. Rev. D. **52**, 7333 (1995)
30. Dray, T., Manogue, C.A., Tucker, R.W.: Phys. Rev. D. **48**, 2587 (1993)
31. Dray, T., Manogue, C.A., Tucker, R.W.: Gen. Relativ. Gravit. **23**, 967 (1991)
32. Dray, T., Manogue, C.A., Tucker, R.W.: Class. Quantum Gravity **12**, 2767 (1995)
33. Romano, J.D.: Phys. Rev. D **47**, 4328 (1993)
34. Gratus, J., Tucker, R.W.: J. Math. Phys. **36**, 3353 (1995)
35. Gratus, J., Tucker, R.W.: J. Math. Phys. **37**, 6018 (1996)
36. Darabi, F., Setare, M.R.: J. Math. Phys. **47**, 032501 (2006)
37. Setare, M.R., Darabi, F.: Casimir effect for a spherical shell in de Sitter spacetime with signature change. hep-th/0511077
38. Setare, M.R.: Class. Quantum Gravity **18**, 4823 (2001)
39. Saharian, A.A.: Phys. Rev. D **63**, 125007 (2001)
40. Setare, M.R., Mansouri, R.: Class. Quantum Gravity **18**, 2331 (2001)